CONTENTS

GENERAL NOTES

These materials are organised in (roughly) increasing levels of difficulty. Though the order can be changed if your scheme of work, for example, covers multiplying binomials before simple factorisation. However, the order in which the techniques appear should not be changed too much as many techniques depend on prior ones.

The exercises are likewise designed to be done more or less in the order presented, as, in general they become more difficult as they progress. If you are doing revision work you may want to concentrate on the mixed exercises which appear at various points, though for introductory work, the examples are often grouped so that patterns can be noticed by the students. An example of this might be where factorising $x^2 + x - 6$ is followed by $x^2 - x - 6$.

Throughout the booklet, x has been substituted for other letters, so that students don't say things like "I can do it with x but not with a." and other similar gripes. The letters have been made the same if a pattern point is being made, as above.

The answers are given after each exercise (or each pair in some cases) so that students can quickly check their answers after each attempt. This is to avoid the unwelcome outcome where a student will practise the same error over and over again, and become extremely adept at making that error. (Unlearning is much harder than learning!)

There are as many different ways of laying out mathematical calculations as there are teachers, so I have intentionally not suggested methods, though I may add these later in the teacher notes and lesson plans, so that you can encourage your students to do it "the right way".

It is important to note that these exercises are but one resource and it is probable that you will want to use other resources in addition, or turn some of the examples into Tarsia jigsaws. The idea behind these exercises is to give a lot of examples to use, as most text book don't provide enough and many online exercises are created randomly using some algorithm or other and so do not point up the patterns. How you use these exercises is entirely up to you. In addition, the mixed exercises make very good consolidation or assessment materials.

Most of the exercises are fairly self-explanatory, but I have added a few notes below on some of them.

Finally, please do not give this pack in its entirety to your year 7s. They will only become terrified unnecessarily and eventually join the ranks of adults who hate and "can't do" mathematics.

SECTION 1 – EXERCISES 1 - 4 – MULTIPLYING, DIVIDING & COLLECTING TERMS

The first section contains exercises for practice in the four basic operations ($\times \div + -$) with algebraic expressions. It is worth noting that students should get into the habit of re-writing any algebraic term with the letters in alphabetical order, and (of course) the numerical digit(s) before the variable(s). Technically, it is not incorrect to write them differently (*cab* as opposed to *abc*) but it makes later work much easier if alphabetical order is always used.

I would always encourage the students to combine terms using three distinct steps:

1. Sign,
2. Coefficient,
3. Letters.

The internal order is unimportant, but three simple steps are always better than one confused step. Those not familiar with this idea, please refer to "How to Solve it" by Georg Polya, which will teach you how to be a better mathematics teacher, like almost no other.

SECTION 2 – EXERCISES 5 - 9 – WORKING WITH BRACKETS #1

This section deals with single brackets for the most part, though later questions do look at how simple factorisation can be applied to grouped brackets. These questions and/or exercises are marked with an asterisk (*) and can be used to stretch the more able students in your care.

SECTION 3 – EXERCISES 10 – 13 – MULTIPLYING GROUPS OF BRACKETS

This section deals with groups of binomials. Again the examples have been grouped together. So in exercise 10 Q1, a signs are +, in Q2 all signs are −. Don't neglect Q3 which consists of squares written as a single binomial raised to the power 2.

Again, I have not suggested a specific method, though in my experience, and at the risk of losing my readers, FOIL is the best.

In exercise 13, we introduce multiplication of 3 linear factors.

SECTION 4 – EXERCISES 14 – 21 – FACTORISING QUADRATICS

These exercises are also in order of difficulty and grouped according to type. Many GCSE (or equivalent) students will not need to go beyond exercise 16, after which $a > 1$. As with previous exercises there are patterns to be spotted amongst adjacent questions and students should be encouraged to look out for patterns as these work through them.

SECTION 5 – EXERCISES 22 – 23 – COMPLETING THE SQUARE

As we are now well into the year 11 (top set) territory, many students will not need to cover these exercises, though I use them to set would be 'A' level students useful material to use over the summer at the end of year 11.

SECTION 6 – EXERCISES 24 - 27 – ALGEBRAIC FRACTIONS

The final exercises involve simplification and combination of algebraic fractions. I find that 'A' level students who struggle with the course, often have major problems with these techniques, so these also are good for the summer after year 11, but are also useful for students doing additional mathematics courses.

The mantra for all of these problems is "factorise first". If students can get this idea, they will go on to mathematical success.

EXERCISE 1 – MULTIPLYING EXPRESSIONS

This exercise gives practice at multiplying numbers and letters together to produce simplified algebraic terms. Q1 – 6 only use positive numbers.
Q7 introduces directed (negative) numbers.
Q8 introduces indices.
Simplify these expressions.

1. **a** $3 \times d$ **a** $2 \times p$ **c** $5 \times q$
    d $a \times 4$ **e** $w \times 5$ **f** $y \times 8$

2. **a** $c \times d$ **b** $b \times a$ **c** $z \times x$
    d $p \times q$ **e** $r \times s$ **f** $m \times n$

3. **a** $2a \times b$ **b** $3p \times q$ **c** $6s \times t$
    d $i \times 4j$ **e** $n \times 7m$ **f** $u \times 4v$

4. **a** $2p \times 3$ **b** $5q \times 3$ **c** $6c \times 8$
    d $9 \times 7t$ **e** $11 \times 3g$ **f** $6 \times 10j$

5. **a** $2x \times 3y$ **b** $4p \times 2q$ **c** $7i \times 5j$
    d $4f \times 3g$ **e** $7a \times 8b$ **f** $9d \times 6e$

6. **a** $3f \times g \times r$ **b** $u \times 4v \times w$ **c** $k \times h \times 6i$
    d $t \times 2r \times 3s$ **e** $a \times 3c \times 8b$ **f** $5s \times 5r \times 3p$
    g $6h \times 2g \times 4i$ **h** $3n \times 7p \times 2m$ **i** $2g \times 5f \times 6e$
    j $5fg \times 7e \times 2c$ **k** $4mn \times 8jk \times 5pq$ **l** $10x \times 10z \times 3y \times 2w$

7. **a** $-2 \times p$ **b** $q \times -5$ **c** $-2m \times 3n$
    d $-5j \times -k$ **e** $-3g \times -5h$ **f** $9s \times -8t$
    g $x \times -y \times 3z$ **h** $-5pq \times -6ab \times 4cd$ **i** $-q \times 8rs \times -4t$
    j $-4bc \times -6ad \times -2ef$ **k** $9rqs \times -4b \times -a$ **l** $-7de \times -8fg \times -2h$

8. **a** $e \times e$ **b** $t \times t$ **c** $a \times a$
    d $4q \times q$ **e** $3r \times r$ **f** $7c \times c$
    g $s \times 3s$ **h** $5r \times 6r$ **i** $4x \times 3x$
    j $2f \times 3f$ **k** $-y \times y \times 8y$ **l** $g \times -5g \times 2g$

9. **a** $z \times -6z \times 3z$ **b** $-6t \times -5t^2$ **c** $2i \times 4i^3$
    d $5t^2 \times -3t \times -t$ **e** $p^2 \times -3p \times -5p$ **f** $-5v^2 \times -7v^2 \times -2v$
    g $2x^2 \times -8x^2$ **h** $-3q^4 \times -5q^3 \times -6q^2$ **i** $-7p^2q \times -2pq$
    j $-8a^5b \times -6ab^2$ **k** $-9p^3q^2r \times 3pqr$ **l** $6xy^2z \times -2x^3y^3z^5$

EXERCISE 1 – MULTIPLYING EXPRESSIONS – ANSWERS

1.   **a** $3d$ **a** $2p$ **c** $5q$

       d $4a$ **e** $5w$ **f** $8y$

2.   **a** cd **b** ab **c** xz

       d pq **e** rs **f** mn

3.   **a** $2ab$ **b** $3pq$ **c** $6st$

       d $4ij$ **e** $7mn$ **f** $4uv$

4.   **a** $6p$ **b** $15q$ **c** $48c$

       d $63t$ **e** $33g$ **f** $60j$

5.   **a** $6xy$ **b** $8pq$ **c** $35ij$

       d $12fg$ **e** $56ab$ **f** $54de$

6.   **a** $3fgr$ **b** $4uvw$ **c** $6hik$

       d $6rst$ **e** $24abc$ **f** $75prs$

       g $48ghi$ **h** $42mnp$ **i** $60efg$

       j $70cefg$ **k** $160jkmnpq$ **l** $600wxyz$

7.   **a** $-2p$ **b** $-5q$ **c** $-6mn$

       d $5jk$ **e** $15gh$ **f** $-72st$

       g $-3xyz$ **h** $120abcdpq$ **i** $32qrst$

       j $-48abcdef$ **k** $36abqrs$ **l** $-112defgh$

8.   **a** e^2 **b** t^2 **c** a^2

       d $4q^2$ **e** $3r^2$ **f** $7c^2$

       g $3s^2$ **h** $30r^2$ **i** $12x^2$

       j $6f^2$ **k** $-8y^3$ **l** $-10g^3$

9.   **a** $-18z^3$ **b** $30t^3$ **c** $8i^4$

       d $15t^4$ **e** $15p^4$ **f** $-70v^5$

       g $-16x^4$ **h** $-90q^9$ **i** $14p^3q^2$

       j $48a^6b^3$ **k** $-27p^4q^3r^2$ **l** $-12x^4y^5z^6$

EXERCISE 2 – DIVIDING EXPRESSIONS

This exercise gives practice at dividing numbers and letters and writing the result as a fraction.
Q4 introduces negative terms.
Q5 introduces indices as well.

Write the following expressions without the ÷ sign and simplify them where possible.

1. **a** $3a \div 3$ **b** $4h \div 2$ **c** $8p \div 4$

     d $27d \div 3$ **e** $35e \div 7$ **f** $48g \div 6$

2. **a** $a \div a$ **b** $c \div c$ **c** $2x \div x$

     d $5w \div w$ **e** $12y \div 2y$ **f** $25x \div 5x$

3. **a** $6s \div 18s$ **b** $3t \div 12t$ **c** $12pq \div 2p$

     d $3s \div 15s$ **e** $20ab \div 4b$ **f** $35dc \div 5cd$

4. **a** $4x \div {}^-2$ **b** ${}^-5t \div {}^-t$ **c** ${}^-6p \div 3p$

     d $6pq \div 42qp$ **e** $mn \div 2nm$ **f** $2rt \div 2tr$

5. **a** $16efg \div 8ef$ **b** $30hij \div -5hj$ **c** ${}^-6zy \div 5zx$

     d ${}^-6cd \div -9de$ **e** $10ab \div {}^-15bc$ **f** ${}^-6xzy \div 14xyz$

6. **a** $g^2 \div g^2$ **b** $e^2 \div e^2$ **c** $a^2 \div a$

     d $2t^2 \div t$ **e** $i^2 \div i$ **f** $y^3 \div y^2$

7. **a** $2b^3 \div 2b$ **b** $3c^3 \div 2c^2$ **c** $u^3 \div 2u^2$

     d ${}^-m^5 \div m^3$ **e** $5j^3 \div {}^-j^2$ **f** ${}^-6y^3 \div {}^-3y$

8. **a** ${}^-8p^4 \div {}^-4p^3$ **b** ${}^-10k^3 \div 2k$ **c** $pq^2 \div {}^-pq$

     d $x^3y^2 \div {}^-xy$ **e** ${}^-m^3n^4 \div mn^2$ **f** $p^2q^2r^3 \div pqr$

9. **a** $4u^7v^5w^4 \div 2wvu$ **b** $24i^3j^3k^3 \div {}^-6i^2j^2k^2$ **c** ${}^-16a^3b^3c^3 \div 8a^2b^2c^2$

     d $6x^5y^3 \div 3x^5y^2$ **e** $20p^2q^5r^3 \div {}^-4pq^2r$ **f** ${}^-15e^4f^5g^6 \div {}^-3e^2g^4f^3$

MANIPULATING ALGEBRAIC EXPRESSIONS

EXERCISE 2 – DIVIDING EXPRESSIONS – ANSWERS

Write the following expressions without the ÷ sign and simplify them where possible.

1. **a** a　　　　　　　**b** $2h$　　　　　　　**c** $2p$
    d $9d$　　　　　　**e** $5e$　　　　　　**f** $8g$

2. **a** 1　　　　　　　**b** 1　　　　　　　**c** 2
    d 5　　　　　　　**e** 6　　　　　　　**f** 5

3. **a** $\frac{1}{3}$　　　　　　　**b** $\frac{1}{4}$　　　　　　　**c** $6q$
    d $\frac{1}{5}$　　　　　　　**e** $5a$　　　　　　　**f** 7

4. **a** $-2x$　　　　　　**b** 5　　　　　　　**c** -2
    d $\frac{1}{7}$　　　　　　　**e** $\frac{1}{2}$　　　　　　　**f** 1

5. **a** $2g$　　　　　　　**b** $-6i$　　　　　　**c** $-\frac{6y}{5x}$
    d $\frac{2c}{3e}$　　　　　　**e** $-\frac{2a}{3c}$　　　　**f** $-\frac{3}{7}$

6. **a** 1　　　　　　　**b** 1　　　　　　　**c** a
    d $2t$　　　　　　　**e** i　　　　　　　**f** y

7. **a** b^2　　　　　　　**b** $\frac{3c}{2}$　　　　　　**c** $\frac{u}{2}$
    d $-m^2$　　　　　　**e** $-5j$　　　　　　**f** $2y^2$

8. **a** $2p$　　　　　　　**b** $-5k^2$　　　　　**c** $-q$
    d $-x^2y$　　　　　**e** $-m^2n^2$　　　　**f** pqr^2

9. **a** $2u^6v^4w^3$　　　　**b** $-4ijk$　　　　　**c** $-2abc$
    d $2y$　　　　　　　**e** $-5pq^3r^2$　　　　**f** $5e^2f^2g^2$

EXERCISE 3 – TERMS

Identify the terms in the following expressions by putting a box around each one. Remember that each term includes the sign before it. It is not necessary to simplify the expressions, but try to identify those which are **like terms**.

1. **a** $a+b-1$
    b $3d-e-f$
    c $7g+3h-2i$

    d $4j-k+3$
    e $m+n-m$
    f $2p+3p-4p$

2. **a** $a+b+c-2$
    b $q-q+q$
    c $rs+2rs-st$

    d $2uv+5vw-4wx$
    e $mn+pq-7$
    f $4st-st+8$

    g b^3-4b^2+b-5
    h $3y^2-2yz+5y$
    i $10a^3-3a^2-4a$

Box the terms in these expressions and write the following expressions as simply as possible.

3. **a** $k+k$
    b $a+a+a$
    c $y+y-y$

    d $2p+p$
    e $5m-m$
    f $4n+2n$

    g $2e+5e$
    h $5x-2x$
    i $3y-7y$

    j $3z-4z$
    k $2a+3b-a-3b$
    l $5q-4p+2q+p$

4. **a** $4e+3ef+3e-2ef$
    b $3i+j-i+4j$
    c $m+n-m+n$

    d $4cd+2dc$
    e $3ab-ba$
    f $5xyz-2zyx$

    g $2ab-4+3ba-2$
    h $2g-2fg+gf-2g$
    i $5c-3dc+5cd-2c$

    j $4mn-4m+3nm-2m$
    k $2r+3s+4t-s+r-t$
    l $2xy+ba-2yx-ab$

5. **a** j^2+j^2
    b s^2+s
    c $2b^2+3b^2$

    d a^2-a^2
    e t^2-2t^2
    f c^2-c-c^2

    g $e^3-e^3+e^3$
    h g^4+2g^4
    i $2j^2-j^2+j-j$

    j $x^2+2x+3x^2+3x$
    k v^2-2v+v^2+3v
    l $4p^2+3p-2p^2-p$

6. **a** $h^2-2g^2+2h^2-g^2$
    b $5x-2x^2+x^2-2x$
    c $u+3v-3u+v^2$

    d $(3xy)^2+4(yx)^2$
    e $5(ef)^3-(3fe)^3$
    f $2a^2bc+3bca^2$

    g $2c^3d+3c-3c-dc^3$
    h $5f^2+2g^3+2f^2-2g^3$
    i $2x^2+x-2+3x^2-4x+5$

    j $7i^2j+3i^2-4ji^2+i^2$
    k $m^2n^3p-n(mn)^2p$
    l $(3sq)^2r+(4q^2r)s^2$

Exercise 3 – Terms – Answers

1.
  a $+a, +b, -1$ **b** $+3d, -e, -f$ **c** $+7g, +3h, -2i$

  d $+4j, -k, +3$ **e** $+m, +n, -m$ **f** $+2p, +3p, -4p$

2.
  a $+a, +b, +c, -2$ **b** $+q, -q, +q$ **c** $+rs, +2rs, -st$

  d $+2uv, +5vw, -4wx$ **e** $+mn, +pq, -7$ **f** $+4rs, -st, +8$

  g $+b^3, -4b^2, +b, -5$ **h** $+3y^2, -2yz, +5y$ **i** $+10a^{3,} -3a^2, -4a$

3.
  a $2k$ **b** $3a$ **c** y

  d $3p$ **e** $4m$ **f** $6n$

  g $7e$ **h** $3x$ **i** $-4y$

  j $-z$ **k** a **l** $7q - 3p$

4.
  a $7e + ef$ **b** $2i + 5j$ **c** $2n$

  d $6cd$ **e** $2ab$ **f** $3xyz$

  g $5ab - 6$ **h** $-fg$ **i** $3c + 2cd$

  j $7mn - 6m$ **k** $3r + 2s + 3t$ **l** 0

5.
  a $2j^2$ **b** $s^2 + s$ **c** $5b^2$

  d 0 **e** $-t^2$ **f** $-c$

  g e^3 **h** $3g^4$ **i** j^2

  j $4x^2 + 5x$ **k** $2v^2 + v$ **l** $2p^2 + 2p$

6.
  a $3h^2 - 3g^2$ **b** $3x - x^2$ **c** $v^2 + 3v - 2u$

  d $13x^2y^2$ **e** $-22e^3f^3$ **f** $5a^2bc$

  g c^3d **h** $7f^2$ **i** $5x^2 - 3x + 3$

  j $4i^2 + 3i^2j$ **k** 0 **l** $13q^2rs^2$

MANIPULATING ALGEBRAIC EXPRESSIONS

EXERCISE 4 – MIXED EXERCISE ON MULTIPLYING, DIVIDING & COLLECTING LIKE TERMS

This is a consolidation exercise which could be set as a homework

Simplify the following expressions.

1. **a** $5 \times t$ **b** $s \div 3$ **c** $uv \times -4$

     d $q + q$ **e** $m + 2m$ **f** $-pr \div r$

2. **a** $a \div ab$ **b** $c \times -c$ **c** $4g \times h$

     d $4d - d$ **e** $2 \div f$ **f** $8j - 3j$

3. **a** $k + k - 2k$ **b** $2j \times 3j$ **c** $6n - 7n$

     d $3y^2 \times -3y$ **e** $4x^2 \div 8x$ **f** $6ei \times -6ei$

4. **a** $2w \div 3w^2$ **b** $4c^2 + 5c^2$ **c** $8z^2 \div 24z^3$

     d $(vw)^2 \times 3wv$ **e** $-18e^3 \div 12e^2f$ **f** $3pq + 20p^2 - 3qp - (4p)^2$

5. **a** $yz + 2zy + z$ **b** $3 \div p \times -q$ **c** $2 \times (a \times b) + 2a \times b$

     d $8z^3 \div (-2z)^3$ **e** $(5st)^2 - (2ts)^2$ **f** $d - (a - 3d)$

6. **a** $3x + x \times (2x - 3)$ **b** $2g - (4h + 2g) - g + 8h$ **c** $x^2 \times (y + 3z) \div x - x \times y$

     d $x - 2 \times (4 - x)$ **e** $c \times (d - e) - ec$ **f** $pq - q \times q + 2q^2$

EXERCISE 4 – MIXED EXERCISE ON MULTIPLYING, DIVIDING & COLLECTING LIKE TERMS – ANSWERS

1. **a** $5t$ **b** $\frac{s}{3}$ **c** $-4uv$ **d** $2q$ **e** $3m$ **f** $-p$

2. **a** $\frac{1}{b}$ **b** $-c^2$ **c** $4gh$ **d** $3d$ **e** $\frac{2}{f}$ **f** $5j$

3. **a** 0 **b** $6j^2$ **c** $-n$ **d** $-9y^3$ **e** $\frac{x}{2}$ **f** $-36e^2i^2$

4. **a** $\frac{2}{3w}$ **b** $9c^2$ **c** $\frac{1}{3z}$ **d** $3v^3w^3$ **e** $-\frac{3e}{2f}$ **f** $4p^2$

5. **a** $3yz + z$ **b** $-\frac{3q}{p}$ **c** $4ab$ **d** -1 **e** $21s^2t^2$ **f** $4d - a$

6. **a** $2x^2$ **b** $4h - g$ **c** $3xz$ **d** $3x - 8$ **e** $cd - 2ce$ **f** $pq + q^2$

EXERCISE 5 – MULTIPLYING A BRACKET BY A SINGLE TERM

From Q4, minus signs are introduced outside the bracket.
From Q5, indices are introduced.
In Q9 you should collect like terms and simplify your answers.

Expand these expressions by removing the brackets.

1. **a** $2(x + 1)$ **b** $3(b + 1)$ **c** $5(q + 2)$

    d $4(y + 3)$ **e** $2(a + 10)$ **f** $7(b + 3)$

2. **a** $8(d - 1)$ **b** $7(c - 5)$ **c** $6(e - 3)$

    d $7(g - 2)$ **e** $9(s - 6)$ **f** $12(t - 12)$

3. **a** $2(3q + 1)$ **b** $8(3p + 7)$ **c** $6(4g + 5)$

    d $3(2w - 5)$ **e** $5(7y - 1)$ **f** $6(9c - 1)$

4. **a** $-2(f + 2)$ **b** $-3(n + 3)$ **c** $-4(4x + 1)$

    d $-1(2p + 1)$ **e** $-(q + 3)$ **f** $-(4j - 3)$

5. **a** $-(-k - 1)$ **b** $-3(5 - r)$ **c** $-10(4m + 1)$

    d $-5(-p + 4)$ **e** $-(x - 3)$ **f** $-6(2n + 3)$

6. **a** $e(e - 1)$ **b** $k(k + 2)$ **c** $q(p - 1)$

    d $a(e + f)$ **e** $h(3j + 2)$ **f** $m(3n - 4)$

7. **a** $c(4d - b)$ **b** $2g(3h + 4)$ **c** $5r(5s - 6)$

    d $p(-t - u)$ **e** $2v(-h + 5v)$ **f** $6q(4 - q)$

8. **a** $7y(2 - 3y)$ **b** $4f(2f + 5)$ **c** $2p(p + 5)$

    d $5y(y + 2)$ **e** $6p(7p^2 - 5q)$ **f** $-s(3s + 4t)$

9*. **a** $-d(-e - 9d^2)$ **b** $t^2(4t^2 - 3t)$ **c** $-x^2(3x^2 + 8x)$

    d $x^2y(8x + y)$ **e** $-n^2(2n - 3)$ **f** $-p^2(3p + 4p^2)$

10*. **a** $2(b + 1) + (b + 2)$ **b** $2(w + 2) + 3(w + 3)$ **c** $3(s + 5) + 2(s + 3)$

    d $4(2m + 3) + 5(3m + 2)$ **e** $2(2r - 3) + 4(3r - 2)$ **f** $6(3q - 5) + 2(4q - 3)$

11*. **a** $3(3b + 4) - 2(2b + 3)$ **b** $5(4p + 7) - 3(3p + 4)$ **c** $6(2j - 5) - (4j + 1)$

    d $5(4q + 5) - 3(5q - 1)$ **e** $5a(a - 3) - 2a(2a - 5)$ **f** $7b(2b - 3c) - 3c(2c - 4b)$

EXERCISE 5 – MULTIPLYING BRACKETS BY A SINGLE TERM – ANSWERS

1. **a** $2x + 2$ **b** $3b + 3$ **c** $5q + 10$
  d $4y + 12$ **e** $2a + 20$ **f** $7b + 21$

2. **a** $8d - 8$ **b** $7c - 35$ **c** $6e - 18$
  d $7g - 14$ **e** $9s - 54$ **f** $12t - 144$

3. **a** $6q + 2$ **b** $24p + 56$ **c** $24g + 30$
  d $6w - 15$ **e** $35y - 5$ **f** $54c - 6$

4. **a** $-2f - 4$ **b** $-3n - 9$ **c** $-16x - 4$
  d $-2p - 1$ **e** $-q - 3$ **f** $-4j + 3$

5. **a** $k + 1$ **b** $-15 + 3r$ **c** $-40m - 10$
  d $5p - 20$ **e** $3 - x$ **f** $-12n - 18$

6. **a** $e^2 - e$ **b** $k^2 + 2k$ **c** $pq - q$
  d $ae + af$ **e** $3hj + 2h$ **f** $3mn - 4m$

7. **a** $4cd - bc$ **b** $6gh + 8g$ **c** $25rs - 30r$
  d $-pt - pu$ **e** $-2hv + 10v^2$ **f** $24q - 6q^2$

8. **a** $14y - 21y^2$ **b** $8f^2 + 20f$ **c** $2p^2 + 10p$
  d $5y^2 + 10y$ **e** $42p^3 - 30pq$ **f** $-3s^2 - 4st$

9*. **a** $de + 9d^3$ **b** $4t^4 - 3t^3$ **c** $-3x^4 - 8x^3$
  d $8x^3y + x^2y^2$ **e** $-2n^3 + 3n^2$ **f** $-3p^3 - 4p^4$

10*. **a** $3b + 4$ **b** $5w + 13$ **c** $5s + 21$
  d $23m + 32$ **e** $16r - 14$ **f** $26q - 36$

11*. **a** $5b + 6$ **b** $11p + 23$ **c** $8j - 31$
  d $5q + 28$ **e** $a^2 - 5a$ **f** $4b^2 - 9bc - 6c^2$

EXERCISE 6 – MULTIPLYING A BRACKET BY A SINGLE TERM (SHORT)

This is a consolidation exercise which could be set as a homework

Expand these expressions by removing the brackets.

1. **a** $5(x+3)$ **b** $3(b-1)$ **c** $7(q-3)$

    d $4(y-3)$ **e** $5(a+b)$ **f** $6(b-4)$

2. **a** $d(d-1)$ **b** $c(2+c)$ **c** $e(e-3)$

    d $2p(p+5)$ **e** $5y(y+2)$ **f** $4f(2f+5)$

    g $5w(w^2+5)$ **h** $11t(3-5t)$ **i** $6q(4-q)$

3. **a** $n^2(2n+3)$ **b** $p^2(3p+4)$ **c** $t^2(4t^2-3t)$

    d $x^2y(8x+y)$ **e** $2abc(3a+2b+c)$ **f** $3de(3d+e+2)$

    g $-(3x+4)$ **h** $-(5r-6)$ **i** $-(10-t)$

4. **a** $7(a+3)+4(a-3)$ **b** $8(6p-5)-3(2p-5)$ **c** $-2r(r+3)-(2-r)$

EXERCISE 7 - FACTORISING SIMPLE EXPRESSIONS

Q5 introduces indices for the first time.
Factorise these expressions fully.

1. **a** $4e+4$ **b** $2y+6$ **c** $9q+3$

    d $3b+3$ **e** $5d+10$ **f** $4s+6$

2. **a** $3b-3$ **b** $2c-10$ **c** $6p-6$

    d $2m-4$ **e** $6c-3$ **f** $10p-8$

3. **a** $pq+2p$ **b** $2s+st$ **c** $ab-bc$

    d $de-2df$ **e** $3np-9p$ **f** $3gh+6g$

4. **a** $4pq-8p$ **b** $8tu+4t$ **c** $4xy-2x+6xz$

    d $14xyz-7xy$ **e** $20uvw-2uv+4vw$ **f** $15gte+5gt-10g$

5. **a** r^2+4r **b** a^2+3a **c** y^2+2y

    d $4t^2+6$ **e** $2s-4s^2$ **f** $6q^2-4q$

    g $2b^2+6b$ **h** $14e^3-21e$ **i** $15g^2+9g$

6*. **a** $3m^2+9m$ **b** $4p^3+9p^2$ **c** $-2x^2+4xy$

    d $-6tu^2-8t$ **e** $-2h-h^2$ **f** $6rs^2t-9rs$

    g $6t^2u-8tu^2$ **h** $18s^3t^2-12st$ **i** $14a^2b+21ab$

    j $-15pq^2+10qp$ **k** $5s-10s^2-15s^4$ **l** $7uv-7u^2v$

    m $(p^2q)^2-p^2q$ **n** $(3a^2b)^2+18a^3b^2$ **o** $6xy^2z-12x^2yz+4xyz^2$

EXERCISE 6 – MULTIPLYING BRACKETS BY A SINGLE TERM – ANSWERS

1. **a** $5x + 15$ **b** $3b - 3$ **c** $7q - 21$

    d $4y - 12$ **e** $5a + 5b$ **f** $6b - 24$

2. **a** $d^2 - d$ **b** $2c + c^2$ **c** $e^2 - 3e$

    d $2p^2 + 10p$ **e** $5y^2 + 10y$ **f** $8f^2 + 20f$

    g $5w^3 + 25w$ **h** $33t - 55t^2$ **i** $24q - 6q^2$

3 **a** $2n^3 + 3n^2$ **b** $3p^3 + 4p^2$ **c** $4t^4 - 3t^3$

    d $8x^3y + x^2y^2$ **e** $6a^2bc + 4ab^2c + 2abc^2$ **f** $9d^2e + 3de^2 + 6de$

    g $-3x - 4$ **h** $-5r + 6$ **i** $-10 + t$

4. **a** $11a + 9$ **b** $42p - 25$ **c** $-r - 8$

EXERCISE 7 – FACTORISING SIMPLE EXPRESSIONS – ANSWERS

1. **a** $4(e + 1)$ **b** $2(y + 3)$ **c** $3(3q + 1)$

    d $3(b + 1)$ **e** $5(d + 2)$ **f** $2(2s + 3)$

2. **a** $3(b - 1)$ **b** $2(c - 5)$ **c** $6(p - 1)$

    d $2(m - 2)$ **e** $3(2c - 1)$ **f** $2(5p - 4)$

3. **a** $p(q + 2)$ **b** $s(2 + t)$ **c** $b(a - c)$

    d $d(e - 2f)$ **e** $3p(n - 3)$ **f** $3g(h + 2)$

4. **a** $4p(q - 2)$ **b** $4t(2u + 1)$ **c** $2x(2y - 1 + 3z)$

    d $7xy(2z - 1)$ **e** $2v(10uw - u + 2w)$ **f** $5g(3et + t - 2)$

5. **a** $r(r + 4)$ **b** $a(a + 3)$ **c** $y(y + 2)$

    d $2(2t^2 + 3)$ **e** $2s(1 - 2s)$ **f** $2q(3q - 2)$

    g $2b(b + 3)$ **h** $7e(2e^2 - 3)$ **i** $3g(5g + 3)$

6*. **a** $3m(m + 3)$ **b** $p^2(4p + 9)$ **c** $2x(2y - x)$

    d $-2t(3u^2 + 4)$ **e** $-h(2 + h)$ **f** $3rs(2st - 3)$

    g $2tu(3t - 4u)$ **h** $6st(3s^2t - 2)$ **i** $7ab(2a + 3)$

    j $5pq(2 - 3q)$ **k** $5s(1 - 2s - 3s^3)$ **l** $7uv(1 - u)$

    m $p^2q(p^2q - 1)$ **n** $9a^3b^2(a + 2)$ **o** $2xyz(3y - 6x + 2z)$

EXERCISE 8* – FACTORISING BY GROUPING

Factorise these expressions fully.

1. **a** $2(a + b) + (a + b)$ **b** $3(c - 2d) + e(c - 2d)$ **c** $g(3m - n) - (3m - n)$

    d $6(p + 3) - 2(p + 3)$ **e** $3(2t + 3) + t^2(2t + 3)$ **f** $(3st + t^2) + 4(3s + t)$

    g $4(5x + 1)^2 - 2(5x + 1)$ **h** $(m + p)^2 + (m + p)$ **i** $(x + y)^2 - 3(x + y)$

2. **a** $(q + 2r) + 3(q + 2r)^2$ **b** $(x^2 + y)^3 - 4(x^2 + y)^2$ **c** $(3m - 2n)^2 - (3m - 2n)^3$

    d $(a - b) - c(a - b)$ **e** $2(m^2 - n) - 3n(n - m^2)$ **f** $(x - 2y) - x(2y - x)$

    g $a(2b - c) + 3(c - 2b)$ **h** $(i - k)^2 - (k - i)$ **i** $(2x - y)^2 - (2y - 4x)$

3. **a** $xy + x + y + 1$ **b** $de + 3e + df + 3f$ **c** $abc - 4ab + 3c - 12$

    d $p^2q + pq + 2p + 2$ **e** $3m^2 + mp + 6mn + 2np$ **f** $i^2jk - ij^2 + ik^2 - jk$

EXERCISE 9 - SIMPLE FACTORISING (MIXED EXERCISE)

Factorise these expressions fully.

1. **a** $12t + 4$ **b** $6x - 3$ **c** $5s - 5$

    d $ab - b$ **e** $dc^2 - c$ **f** $2d + 5de$

    g $3 - 6g^2$ **h** $10rs + 2s$ **i** $15pq + 10q^2$

2. **a** $8x^2 - 4xy$ **b** $15p^3 - 10p$ **c** $7rst + 7r$

    d $(2b^2)^3 + b^5$ **e** $6abc + 9ab - 15bc$ **f** $3ab^2 - 6ab + 9b$

3. **a** $2\pi rh + \pi r^2$ **b** $\frac{2}{3}\pi r^2 + \frac{1}{3}\pi r^3$ **c** $-st - 3t$

    d $-4d^2 - 6de$ **e** $a(b + c) + d(b + c)$ **f** $2x(x^2 - 3) - 3(x^2 - 3)$

    g $t(1 - t) - (t - 1)$ **h** $(g + h)^3 - (g + h)^2$ **i** $6(p - 3q) - 4(6q - 2p)$

EXERCISE 8* – FACTORISING BY GROUPING – ANSWERS

1. **a** $3(a + b)$ **b** $(3 + e)(c - 2d)$ **c** $(g - 1)(3m - n)$

     d $4(p + 3)$ **e** $(3 + t^2)(2t + 3)$ **f** $(t + 4)(3s + t)$

     g $2(5x + 1)(10x + 1)$ **h** $(m + p)(m + p + 1)$ **i** $(x + y)(x + y - 3)$

2. **a** $(q + 2r)(1 + 3q + 6r)$ **b** $(x^2 + y)^2(x^2 + y - 4)$ **c** $(3m - 2n)^2(1 - 3m + 2n)$

     d $(a - b)(1 - c)$ **e** $(2 + 3n)(m^2 - n)$ **f** $(1 + x)(x - 2y)$

     g $(a - 3)(2b - c)$ **h** $(i - k)(i - k + 1)$ **i** $(2x - y)(2x - y + 2)$

3. **a** $(x + 1)(y + 1)$ **b** $(e + f)(d + 3)$ **c** $(c - 4)(ab + 3)$

     d $(p + 1)(pq + 2)$ **e** $(m + 2n)(3m + p)$ **f** $(ij + k)(ik - j)$

EXERCISE 9 – SIMPLE FACTORISING (MIXED EXERCISE) – ANSWERS

1. **a** $4(3t + 1)$ **b** $3(2x - 1)$ **c** $5(s - 1)$

     d $b(a - 1)$ **e** $c(cd - 1)$ **f** $d(2 + 5e)$

     g $3(1 - 2g^2)$ **h** $2s(5r + 1)$ **i** $5q(3p + 2q)$

2. **a** $4x(2x - y)$ **b** $5p(3p^2 - 2)$ **c** $7r(st + 1)$

     d $b^5(8b + 1)$ **e** $3b(2ac + 3a - 5c)$ **f** $3b(ab - 2a + 3)$

3. **a** $\pi r(2h + r)$ **b** $\dfrac{\pi r^2}{3}(2 + r)$ **c** $-t(s + 3)$

     d $-2d(2d + 3e)$ **e** $(a + d)(b + c)$ **f** $(2x - 3)(x^2 - 3)$

     g $(1 - t)(1 + t)$ **h** $(g + h)^2(g + h - 1)$ **i** $14(p - 3q)$

EXERCISE 10 – MULTIPLYING PAIRS OF BRACKETS (SIGNS THE SAME)

Hint: *The parts of questions* 1 *and* 2 *are very similar. See if you can spot the pattern.*

Q4 *is a mixed of the types we have met in* Q1–3

Multiply out the following pairs of brackets and then simplify your answer if possible.

1. **a** $(f+1)(f+2)$ **b** $(a+1)(a+3)$ **c** $(m+2)(m+3)$

d $(r+2)(r+4)$ **e** $(g+2)(g+5)$ **f** $(b+3)(b+4)$

g $(h+3)(h+5)$ **h** $(n+4)(n+5)$ **i** $(v+6)(v+2)$

j $(c+7)(c+3)$ **k** $(s+6)(s+4)$ **l** $(p+8)(p+3)$

m $(d+5)(d+8)$ **n** $(e+8)(e+9)$ **o** $(i+7)(i+9)$

2. **a** $(x-1)(x-2)$ **b** $(i-1)(i-3)$ **c** $(z-2)(z-3)$

d $(j-2)(j-4)$ **e** $(d-2)(d-5)$ **f** $(t-3)(t-4)$

g $(q-3)(q-5)$ **h** $(w-4)(w-5)$ **i** $(y-6)(y-2)$

j $(u-7)(u-3)$ **k** $(k-6)(k-4)$ **l** $(e-8)(e-3)$

m $(a-5)(a-8)$ **n** $(b-8)(b-9)$ **o** $(g-7)(g-9)$

3. **a** $(f+3)^2$ **b** $(k-3)^2$ **c** $(a+2)^2$

d $(q-2)^2$ **e** $(v+5)^2$ **f** $(g-5)^2$

g $(b+7)^2$ **h** $(m-7)^2$ **i** $(r+6)^2$

j $(h-6)^2$ **k** $(z+9)^2$ **l** $(w-9)^2$

4*. **a** $(a+3)(a+7)$ **b** $(x-3)^2$ **c** $(y-3)(y-9)$

d $(b+6)(b+5)$ **e** $(w-10)(w-9)$ **f** $(s+11)(s+9)$

g $(m+n)^2$ **h** $(a-b)^2$ **i** $(t+3)(3+t)$

j $(2-r)(r-3)$ **k** $(q-3)(3-q)$ **l** $(p-q)(q-p)$

Exercise 10 – Multiplying pairs of brackets #1 – Answers

1.   **a** $f^2 + 3f + 2$ **b** $a^2 + 4a + 3$ **c** $m^2 + 5m + 6$

     d $r^2 + 6r + 8$ **e** $g^2 + 7g + 10$ **f** $b^2 + 7b + 12$

     g $h^2 + 8h + 15$ **h** $n^2 + 9n + 20$ **i** $v^2 + 8v + 12$

     j $c^2 + 10c + 21$ **k** $s^2 + 10s + 24$ **l** $p^2 + 11p + 24$

     m $d^2 + 13d + 40$ **n** $e^2 + 17e + 72$ **o** $i^2 + 16i + 63$

2.   **a** $x^2 - 3x + 2$ **b** $i^2 - 4i + 3$ **c** $z^2 - 5z + 6$

     d $j^2 - 6j + 8$ **e** $d^2 - 7d + 10$ **f** $t^2 - 7t + 12$

     g $q^2 - 8q + 15$ **h** $w^2 - 9w + 20$ **i** $y^2 - 8y + 12$

     j $u^2 - 10u + 21$ **k** $k^2 - 10k + 24$ **l** $e^2 - 11e + 24$

     m $a^2 - 13a + 40$ **n** $b^2 - 17b + 72$ **o** $g^2 - 16g + 63$

3.   **a** $f^2 + 6f + 9$ **b** $k^2 - 6k + 9$ **c** $a^2 + 4a + 4$

     d $q^2 - 4q + 4$ **e** $v^2 + 10v + 25$ **f** $g^2 - 10g + 25$

     g $b^2 + 14b + 49$ **h** $m^2 - 14m + 49$ **i** $r^2 + 12r + 36$

     j $h^2 - 12h + 36$ **k** $z^2 + 18z + 81$ **l** $w^2 - 18w + 81$

4*.   **a** $a^2 + 10a + 21$ **b** $x^2 - 6x + 9$ **c** $y^2 - 12y + 27$

     d $b^2 + 11b + 30$ **e** $w^2 - 19w + 90$ **f** $s^2 + 20s + 99$

     g $m^2 + 2mn + n^2$ **h** $a^2 - 2ab + b^2$ **i** $t^2 + 6t + 9$

     j $^-r^2 + 5r - 6$ **k** $^-q^2 + 6q - 9$ **l** $^-p^2 + 2pq - q^2$

EXERCISE 11 – MULTIPLYING PAIRS OF BRACKETS (DIFFERENT SIGNS)

Q1 has been organised in pairs. Can you find a pattern in the questions and answers.
Q4 introduces difference of two squares. Try to spot the pattern in the questions and answers.
Q5 has slightly more complex examples.

Multiply out the following pairs of brackets and then simplify your answer if possible.

1.
 a $(e-1)(e+2)$
 b $(e+1)(e-2)$
 c $(q-1)(q+3)$

 d $(q+1)(q-3)$
 e $(u-2)(u+4)$
 f $(u+2)(u-4)$

 g $(r-5)(r+3)$
 h $(r+5)(r-3)$
 i $(k-6)(k+2)$

 j $(k+6)(k-2)$
 k $(b-7)(b+5)$
 l $(b+7)(b-5)$

2.
 a $(m-6)(m+7)$
 b $(s-2)(s+8)$
 c $(g-5)(g+2)$

 d $(a+9)(a-8)$
 e $(p+10)(p-5)$
 f $(r-12)(r+11)$

 g $(b-8)(b+11)$
 h $(x+16)(x-5)$
 i $(z-6)(z+8)$

3.
 a $(z-2)(z+9)$
 b $(w-4)(w+7)$
 c $(n-5)(n+8)$

 d $(t-8)(t+7)$
 e $(h-9)(h+3)$
 f $(c-4)(c+10)$

 g $(p-7)(p+9)$
 h $(d-3)(d+7)$
 i $(i-8)(i+9)$

4.
 a $(s-1)(s+1)$
 b $(c-3)(c+3)$
 c $(n-7)(n+7)$

 d $(x+6)(x-6)$
 e $(i+11)(i-11)$
 f $(t+5)(t-5)$

 g $(p-8)(p+8)$
 h $(u-10)(u+10)$
 i $(d-2)(d+2)$

 j $(y+9)(y-9)$
 k $(e-4)(e+4)$
 l $(j+12)(j-12)$

5*.
 a $(u-3)(3+u)$
 b $(b-z)(b+z)$
 c $(hd+5)(hd-5)$

 d $(4+g)(g-4)$
 e $(5+x)(4-x)$
 f $(3-u)(5+u)$

 g $(8-v)(v+6)$
 h $(r-7)(8+r)$
 i $(5-y)(y+3)$

 j $(10+v)(v-11)$
 k $(9-i)(8+z)$
 l $(a+3)(b-4)$

EXERCISE 11 – MULTIPLYING PAIRS OF BRACKETS #2 – ANSWERS

1
a $e^2 + e - 2$ **b** $j^2 - j - 2$ **c** $q^2 + 2q - 3$

d $a^2 - 2a - 3$ **e** $u^2 + 2u - 8$ **f** $x^2 - 2x - 8$

g $r^2 - 2r - 15$ **h** $f^2 + 2f - 15$ **i** $k^2 - 4k - 12$

j $v^2 + 4v - 12$ **k** $b^2 - 2b - 35$ **l** $y^2 + 2y - 35$

2.
a $m^2 - m - 42$ **b** $s^2 + 6s - 16$ **c** $g^2 - 3g - 10$

d $a^2 + a - 72$ **e** $p^2 + 5p - 50$ **f** $r^2 - r - 132$

g $b^2 + 3b - 88$ **h** $x^2 + 11x - 80$ **i** $z^2 + 2z - 48$

3
a $z^2 + 7z - 18$ **b** $w^2 + 3w - 28$ **c** $n^2 + 3n - 40$

d $t^2 - t - 56$ **e** $h^2 - 6h - 27$ **f** $c^2 + 6c - 40$

g $p^2 + 2p - 63$ **h** $d^2 + 4d - 21$ **i** $i^2 + i - 72$

4.
a $s^2 - 1$ **b** $c^2 - 9$ **c** $n^2 - 49$

d $x^2 - 36$ **e** $i^2 - 121$ **f** $t^2 - 25$

g $p^2 - 64$ **h** $u^2 - 100$ **i** $d^2 - 4$

j $y^2 - 81$ **k** $e^2 - 16$ **l** $j^2 - 144$

5*.
a $u^2 - 9$ **b** $b^2 - z^2$ **c** $h^2d^2 - 25$

d $g^2 - 16$ **e** $-x^2 - x + 20$ **f** $-u^2 - 2u + 15$

g $-v^2 + 2v + 48$ **h** $r^2 + r - 56$ **i** $-y^2 + 2y + 15$

j $v^2 - v - 110$ **k** $72 + 9z - 8i - iz$ **l** $ab - 4a + 3b - 12$

EXERCISE 12 – MULTIPLYING BRACKETS (MORE COMPLEX EXAMPLES)

Hint: Remember to multiply out in 3 stages, signs, numbers & letters.

Multiply the following pairs of brackets and then simplify your answer if possible.

1. **a** $(2f+1)(f+1)$ **b** $(4m+3)(m+1)$ **c** $(2w+5)(w+4)$

   d $(2n+3)(2n+3)$ **e** $(3z+2)(2z+3)$ **f** $(3k+4)^2$

2 **a** $(2a-7)(4a-5)$ **b** $(4d-3)(4d-3)$ **c** $(3r-4)(r-1)$

   d $(2s-3)(2s-4)$ **e** $(5t-2)(2t-5)$ **f** $(5-2i)(1-i)$

3. **a** $(2x-5)(3x+2)$ **b** $(2h-3)(2h+3)$ **c** $(5-3j)(5+3j)$

   d $(2+3c)(1-2c)$ **e** $(3+2y)(4y-1)$ **f** $(3a-1)(a+2)$

4. **a** $(2-3p)(p+2)$ **b** $(3q+2v)(3q-2v)$ **c** $(ax+b)(ax-b)$

   d $(4-u)(4+u)$ **e** $(2+p)(2-3p)$ **f** $(ax+b)^2$

EXERCISE 13 – MULTIPLYING 3 LINEAR FACTORS

1. **a** $x(x+1)(x+2)$ **b** $p(p+1)(p+3)$ **c** $a(a+3)(a+4)$

   d $b(b-2)(b-1)$ **e** $d(d-4)(d-5)$ **f** $q(q-4)(q-6)$

2 **a** $v(v-1)(v+1)$ **b** $t(t+1)(t-2)$ **c** $q(q-5)(q+3)$

   d $m(m-2)(m+5)$ **e** $z(z-5)(z+3)$ **f** $r(r-7)(r+3)$

3. **a** $2d(d+3)(d+4)$ **b** $4f(f-2)(f-4)$ **c** $5g(g-2)(g+1)$

   d $6k(1+k)(3-k)$ **e** $2b(2-b)(b+6)$ **f** $3j(3-j)(3+j)$

4. **a** $(a+1)(a+1)(a+1)$ **b** $(s+2)(s+2)(s+2)$ **c** $(r+1)(r+1)(r+2)$

   d $(n+2)(n+3)(n+4)$ **e** $(g+3)(g+5)(g+4)$ **f** $(h+3)(h+5)(h+6)$

5. **a** $3a(2a-3)(3a-2)$ **b** $3t(2t-5)(3t+2)$ **c** $-6y(4y-1)(3y+2)$

   d $(2x+3)(2x-1)(3x-4)$ **e** $(2u-5)(3u+2)(4u-3)$ **f** $(3c+7)(c-2)(5c-3)$

*6. **a** $(3p-4)(3p-2)(3p^2-1)$ **b** $(a^2+4)(2a-3)(a+2)$ **c** $(3b^2+4)(4b^2-3)(3b^2+1)$

   d $(1-c^2)(2-c^2)(3-c^2)$ **e** $(2-g^2)(3-g^2)(3+2g^2)$ **f** $(3-m^2)(2-3m^2)(2m^2-5)$

EXERCISE 12 – MULTIPLYING PAIRS OF BRACKETS #3 – ANSWERS

1.   **a** $2f^2 + 3f + 1$ **b** $4m^2 + 7m + 3$ **c** $2w^2 + 13w + 20$

       d $4n^2 + 12n + 9$ **e** $6z^2 + 13z + 6$ **f** $9k^2 + 24k + 16$

2.   **a** $8a^2 - 38a + 35$ **b** $16d^2 - 24d + 9$ **c** $3r^2 - 7r + 4$

       d $4s^2 - 14s + 12$ **e** $10t^2 - 29t + 10$ **f** $5 - 7i + 2i^2$

3.   **a** $6x^2 - 11x - 10$ **b** $4h^2 - 9$ **c** $25 - 9j^2$

       d $2 - c - 6c^2$ **e** $8y^2 + 10y - 3$ **f** $3a^2 + 5a - 2$

4.   **a** $25 - 9j^2$ **b** $9q^2 - 4v^2$ **c** $a^2x^2 - b^2$

       d $16 - u^2$ **e** $4 - 4p - 3p^2$ **f** $a^2x^2 + 2abx + b^2$

EXERCISE 13 – MULTIPLYING 3 LINEAR FACTORS – ANSWERS

1.   **a** $x^3 + 3x^2 + 2x$ **b** $p^3 + 4p^2 + 3p$ **c** $a^3 + 7a^2 + 12a$

      d $b^3 - 3b^2 + 2b$ **e** $d^3 - 9d^2 + 20d$ **f** $q^3 - 10q^2 + 24q$

2   **a** $v^3 - v$ **b** $t^3 - t^2 - 2t$ **c** $q^3 - 2q^2 - 15q$

      d $m^3 + 2m^2 - 10m$ **e** $z^3 - 2z^2 - 15z$ **f** $r^3 - 4r^2 - 21r$

3.   **a** $2d^3 + 14d^2 + 24d$ **b** $4f^3 - 24f^2 + 32f$ **c** $5g^3 - 5g^2 - 10g$

      d $18k + 12k^2 - 6k^3$ **e** $24b - 8b^2 - 2b^3$ **f** $27j - 3j^3$

4.   **a** $a^3 + 3a^2 + 3a + 1$ **b** $s^3 + 6s^2 + 12s + 8$ **c** $r^3 + 4r^2 + 5r + 2$

      d $n^3 + 9n^2 + 26n + 24$ **e** $g^3 + 12g^2 + 47g + 60$ **f** $h^3 + 14h^2 + 63h + 90$

5.   **a** $18a^3 - 39a^2 + 18a$ **b** $18t^3 - 33t^2 - 30t$ **c** $-72y^3 - 30y^2 + 12y$

      d $12x^3 - 4x^2 - 25x + 12$ **e** $24u^3 - 62u^2 - 7u + 30$ **f** $15c^3 - 4c^2 - 73c + 42$

6.   **a** $27p^4 - 54p^3 + 15p^2 + 18p - 8$ **b** $2a^4 + a^3 + 2a^2 + 4a - 24$ **c** $36b^6 - 11b^4 - 29b^2 - 12$

      d $6 - 11c^2 + 6c^4 - c^6$ **e** $18 - 3g^2 - 7g^4 + 2g^6$ **f** $-30 + 77m^2 - 56m^4 + 12m^6$

EXERCISE 14 – FACTORISING QUADRATIC EXPRESSIONS #1 ($a = 1$ & $b, c > 0$)

Hint: All of these quadratic expressions can be factorised. Try to spot patterns between examples.

Factorise the following expressions.

1.
   a $x^2 + 3x + 2$
   b $n^2 + 2n + 1$
   c $a^2 + 4a + 3$

   d $p^2 + 6p + 5$
   e $m^2 + 5m + 4$
   f $q^2 + 8q + 7$

   g $s^2 + 10s + 9$
   h $g^2 + 7g + 6$
   i $h^2 + 5h + 6$

2.
   a $u^2 + 13u + 12$
   b $k^2 + 8k + 12$
   c $r^2 + 7r + 12$

   d $f^2 + 6f + 8$
   e $w^2 + 8w + 15$
   f $v^2 + 16v + 15$

   g $p^2 + 9p + 18$
   h $m^2 + 19m + 18$
   i $a^2 + 11a + 18$

3.
   a $k^2 + 17k + 16$
   b $c^2 + 21c + 20$
   c $q^2 + 12q + 20$

   d $b^2 + 9b + 20$
   e $x^2 + 14x + 24$
   f $s^2 + 11s + 24$

   g $t^2 + 10t + 24$
   h $t^2 + 8t + 16$
   i $m^2 + 9m + 14$

EXERCISE 15 – FACTORISING QUADRATICS #2 ($a = 1$, $b < 0$ & $c > 0$)

Hint: All of these quadratic expressions can be factorised. Again, take your time and try to spot the patterns.

Factorise these expressions.

1.
   a $b^2 - 4b + 4$
   b $x^2 - 2x + 1$
   c $s^2 - 3s + 2$

   d $q^2 - 8q + 16$
   e $c^2 - 5c + 6$
   f $p^2 - 6p + 5$

   g $k^2 - 6k + 8$
   h $m^2 - 5m + 4$
   i $a^2 - 4a + 3$

2.
   a $w^2 - 7w + 12$
   b $v^2 - 6v + 9$
   c $u^2 - 8u + 12$

   d $r^2 - 10r + 21$
   e $f^2 - 9f + 18$
   f $k^2 - 14k + 24$

   g $t^2 - 17t + 16$
   h $h^2 - 9h + 20$
   i $s^2 - 11s + 18$

3.
   a $n^2 - 12n + 20$
   b $p^2 - 11p + 24$
   c $m^2 - 8m + 15$

   d $x^2 - 19x + 18$
   e $a^2 - 16a + 15$
   f $q^2 - 13q + 12$

   g $d^2 - 10d + 24$
   h $g^2 - 25g + 24$
   i $e^2 - 13e + 36$

EXERCISE 14 – FACTORISING QUADRATIC EXPRESSIONS ($a = 1$ & b, $c > 0$) – ANSWERS

1. **a** $(x + 1)(x + 2)$ **b** $(n + 1)(n + 1)$ **c** $(a + 1)(a + 3)$

   d $(p + 1)(p + 5)$ **e** $(m + 1)(m + 4)$ **f** $(q + 7)(q + 1)$

   g $(s + 9)(s + 1)$ **h** $(g + 6)(g + 1)$ **i** $(h + 2)(h + 3)$

2. **a** $(u + 1)(u + 12)$ **b** $(k + 2)(k + 6)$ **c** $(r + 3)(r + 4)$

   d $(f + 2)(f + 4)$ **e** $(w + 3)(w + 5)$ **f** $(v + 1)(v + 15)$

   g $(p + 3)(p + 6)$ **h** $(m + 1)(m + 18)$ **i** $(a + 2)(a + 9)$

3. **a** $(k + 1)(k + 16)$ **b** $(c + 1)(c + 20)$ **c** $(q + 2)(q + 10)$

   d $(b + 4)(b + 5)$ **e** $(x + 2)(x + 12)$ **f** $(s + 3)(s + 8)$

   g $(t + 4)(t + 6)$ **h** $(t + 4)(t + 4)$ **i** $(m + 7)(m + 2)$

EXERCISE 15 – FACTORISING QUADRATICS #2 ($a = 1$, $b < 0$, $c > 0$) – ANSWERS

1. **a** $(b - 2)(b - 2)$ **b** $(x - 1)(x - 1)$ **c** $(s - 1)(s - 2)$

   d $(q - 4)(q - 4)$ **e** $(c - 2)(c - 3)$ **f** $(p - 1)(p - 5)$

   g $(k - 2)(k - 4)$ **h** $(m - 1)(m - 4)$ **i** $(a - 1)(a - 3)$

2. **a** $(w - 3)(w - 4)$ **b** $(v - 3)(v - 3)$ **c** $(u - 2)(u - 6)$

   d $(r - 3)(r - 7)$ **e** $(f - 3)(f - 6)$ **f** $(k - 2)(k - 12)$

   g $(t - 1)(t - 16)$ **h** $(h - 4)(h - 5)$ **i** $(s - 2)(s - 9)$

3. **a** $(n - 2)(n - 10)$ **b** $(p - 3)(p - 8)$ **c** $(m - 3)(m - 5)$

   d $(x - 1)(x - 18)$ **e** $(a - 1)(a - 15)$ **f** $(q - 1)(q - 12)$

   g $(d - 6)(d - 4)$ **h** $(g - 24)(g - 1)$ **i** $(e - 9)(e - 4)$

EXERCISE 16 – FACTORISING QUADRATICS #3 ($a = 1$, $c < 0$)

Hint: All of these quadratic expressions can be factorised. Once again try to spot patterns between the examples.

Factorise the following expressions.

1. **a** $d^2 + d - 2$ **b** $k^2 - k - 2$ **c** $r^2 - r - 6$

d $x^2 + x - 6$ **e** $e^2 + 5e - 6$ **f** $m^2 - 5m - 6$

g $w^2 - 4w - 5$ **h** $z^2 - 2z - 8$ **i** $f^2 + 2f - 8$

2. **a** $u^2 - 49$ **b** $s^2 - 4$ **c** $t^2 - 25$

d $g^2 + 2g - 15$ **e** $a^2 - 2a - 15$ **f** $y^2 + 15y - 16$

g $h^2 + 6h - 16$ **h** $p^2 - 6p - 16$ **i** $n^2 + 3n - 10$

3. **a** $i^2 + 19i - 20$ **b** $v^2 - 19v - 20$ **c** $b^2 + 8b - 20$

d $q^2 - 15q - 16$ **e** $c^2 + c - 20$ **f** $j^2 - j - 20$

g $r^2 + 5r - 24$ **h** $k^2 - 2k - 24$ **i** $z^2 - 23z - 24$

EXERCISE 17 – FACTORISING QUADRATICS #4 (MIXED EXERCISE)

Factorise the following expressions.

1. **a** $d^2 - 2d - 24$ **b** $j^2 + 11j + 30$ **c** $q^2 - 7q - 18$

d $u^2 + 5u - 14$ **e** $w^2 + 5w - 24$ **f** $e^2 - 5e - 24$

g $r^2 - 15r + 14$ **h** $z^2 + 20z + 36$ **i** $k^2 - 6k - 27$

2. **a** $x^2 + 7x + 10$ **b** $f^2 + 23f - 24$ **c** $s^2 + 9s + 18$

d $m^2 + 6m - 7$ **e** $v^2 + 3v - 28$ **f** $y^2 - 2y - 35$

g $a^2 + 13a - 30$ **h** $n^2 - 36$ **i** $g^2 - g - 30$

3. **a** $t^2 + 4t - 32$ **b** $h^2 - 8h - 20$ **c** $b^2 - 3b - 10$

d $p^2 - 16$ **e** $c^2 + 4c - 5$ **f** $i^2 - 14i - 32$

g $d^2 - d - 42$ **h** $e^2 + 2e - 35$ **i** $i^2 + i - 56$

EXERCISE 16 – FACTORISING QUADRATICS #3 ($a = 1$, $c > 0$) – ANSWERS

1. **a** $(d-1)(d+2)$ **b** $(k-2)(k+1)$ **c** $(r-3)(r+2)$

    d $(x-2)(x+3)$ **e** $(e-1)(e+6)$ **f** $(m-6)(m+1)$

    g $(w-5)(w+1)$ **h** $(z-4)(z+2)$ **i** $(f-2)(f+4)$

2. **a** $(u-7)(u+7)$ **b** $(s-2)(s+2)$ **c** $(t-5)(t+5)$

    d $(g-3)(g+5)$ **e** $(a-5)(a+3)$ **f** $(y-1)(y+16)$

    g $(h-2)(h+8)$ **h** $(p-8)(p+2)$ **i** $(n-2)(n+5)$

3. **a** $(i-1)(i+20)$ **b** $(v-20)(v+1)$ **c** $(b-2)(b+10)$

    d $(q-16)(q+1)$ **e** $(c-4)(c+5)$ **f** $(j-5)(j+4)$

    g $(r-3)(r+8)$ **h** $(k-6)(k+4)$ **i** $(z-24)(z+1)$

EXERCISE 17 – FACTORISING QUADRATICS #4 ($a = 1$, $c < 0$) – ANSWERS

1. **a** $(d-6)(d+4)$ **b** $(j+5)(j+6)$ **c** $(q-9)(q+2)$

    d $(u-2)(u+7)$ **e** $(w-3)(w+8)$ **f** $(e-8)(e+3)$

    g $(r-1)(r-14)$ **h** $(z+2)(z+18)$ **i** $(k-9)(k+3)$

2. **a** $(x+2)(x+5)$ **b** $(f-1)(f+24)$ **c** $(s+3)(s+6)$

    d $(m-1)(m+7)$ **e** $(v-4)(v+7)$ **f** $(y-7)(y+5)$

    g $(a-2)(a+15)$ **h** $(n-6)(n+6)$ **i** $(g-6)(g+5)$

3. **a** $(t-4)(t+8)$ **b** $(h-10)(h+2)$ **c** $(b-5)(b+2)$

    d $(p-4)(p+4)$ **e** $(c-1)(c+5)$ **f** $(i-16)(i+2)$

    g $(d-7)(d+6)$ **h** $(e+7)(e-5)$ **i** $(i+8)(i-7)$

EXERCISE 18 – FACTORISING QUADRATICS #5 ($a > 1$)

Factorise the following quadratic expressions.

1. **a** $2f^2 + 3f + 1$ **b** $3a^2 + 4a + 1$ **c** $5m^2 + 6m + 1$

     d $6s^2 + 5s + 1$ **e** $10g^2 + 7g + 1$ **f** $8w^2 + 6w + 1$

     g $3z^2 + 10z + 3$ **h** $6n^2 + 11n + 3$ **i** $6b^2 + 13b + 6$

2. **a** $4t^2 + 7t + 3$ **b** $3h^2 + 7h + 2$ **c** $2p^2 + 5p + 2$

     d $6x^2 + 19x + 3$ **e** $8c^2 + 14c + 3$ **f** $6i^2 + 23i + 20$

     g $9y^2 + 12y + 4$ **h** $4q^2 + 20q + 25$ **i** $9u^2 + 24u + 16$

3. **a** $12d^2 + 17d + 6$ **b** $12j^2 + 28j + 15$ **c** $6r^2 + 19r + 15$

     d $21k^2 + 37k + 12$ **e** $10e^2 + 23e + 12$ **f** $18v^2 + 27v + 10$

     g $12i^2 + 47i + 40$ **h** $35h^2 + 57h + 18$ **i** $9i^2 + 27i + 20$

EXERCISE 19 – FACTORISING QUADRATICS #6 ($a > 1$, $b < 0$, $c > 0$)

Hint: Some of the questions have a numerical factor as well as the algebraic ones.
Factorise the following quadratics fully.

1. **a** $2e^2 - 3e + 1$ **b** $3k^2 - 4k + 1$ **c** $5q^2 - 6q + 1$

     d $3u^2 - 5u + 2$ **e** $6f^2 - 11f + 3$ **f** $4v^2 - 13v + 3$

     g $4m^2 - 8m + 3$ **h** $3r^2 - 10r + 8$ **i** $5g^2 - 16g + 3$

2. **a** $6x^2 - 13x + 6$ **b** $8a^2 - 22a + 15$ **c** $12z^2 - 29z + 15$

     d $6s^2 - 23s + 20$ **e** $12h^2 - 17h + 6$ **f** $20w^2 - 23w + 6$

     g $12b^2 - 23b + 10$ **h** $6n^2 - 16n + 8$ **i** $20y^2 - 22y + 6$

3. **a** $12i^2 - 48i + 21$ **b** $15p^2 - 22p + 8$ **c** $18c^2 - 30c + 8$

     d $10p^2 - 27p + 18$ **e** $21d^2 - 34d + 8$ **f** $20j^2 - 36j + 9$

     g $30k^2 - 47k + 7$ **h** $24x^2 - 62x + 40$ **i** $84x^2 - 188x + 80$

EXERCISE 18 – FACTORISING QUADRATICS #5 ($a > 1$, $c > 0$) – ANSWERS

1. **a** $(2f + 1)(f + 1)$ **b** $(3a + 1)(a + 1)$ **c** $(5m + 1)(m + 1)$

   d $(2s + 1)(3s + 1)$ **e** $(2g + 1)(5g + 1)$ **f** $(4w + 1)(2w + 1)$

   g $(3z + 1)(z + 3)$ **h** $(3n + 1)(2n + 3)$ **i** $(2b + 3)(3b + 2)$

2. **a** $(t + 1)(4t + 3)$ **b** $(3h + 1)(h + 2)$ **c** $(2p + 1)(p + 2)$

   d $(6x + 1)(x + 3)$ **e** $(2c + 3)(4c + 1)$ **f** $(2i + 5)(3i + 4)$

   g $(3y + 2)(3y + 2)$ **h** $(2q + 5)(2q + 5)$ **i** $(3u + 4)(3u + 4)$

3. **a** $(3d + 2)(4d + 3)$ **b** $(6j + 5)(2j + 3)$ **c** $(2r + 3)(3r + 5)$

   d $(7k + 3)(3k + 4)$ **e** $(5e + 4)(2e + 3)$ **f** $(6v + 5)(3v + 2)$

   g $(3i + 8)(4i + 5)$ **h** $(5h + 6)(7h + 3)$ **i** $(3i + 4)(3i + 5)$

EXERCISE 19 – FACTORISING QUADRATICS #6 ($a > 1$, $c < 0$) – ANSWERS

1. **a** $(2e - 1)(e - 1)$ **b** $(3k - 1)(k - 1)$ **c** $(5q - 1)(q - 1)$

   d $(3u - 2)(u - 1)$ **e** $(3f - 1)(2f - 3)$ **f** $(4v - 1)(v - 3)$

   g $(2m - 3)(2m - 1)$ **h** $(r - 2)(3r - 4)$ **i** $(5g - 1)(g - 3)$

2. **a** $(3x - 2)(2x - 3)$ **b** $(4a - 5)(2a - 3)$ **c** $(4z - 3)(3z - 5)$

   d $(3s - 4)(2s - 5)$ **e** $(3h - 2)(4h - 3)$ **f** $(5w - 2)(4w - 3)$

   g $(3b - 2)(4b - 5)$ **h** $2(3n - 2)(n - 2)$ **i** $2(5y - 3)(2y - 1)$

3. **a** $3(2i - 7)(2i - 1)$ **b** $(3p - 2)(5p - 4)$ **c** $2(3c - 1)(3c - 4)$

   d $(2p - 3)(5p - 6)$ **e** $(7d - 2)(3d - 4)$ **f** $(10j - 3)(2j - 3)$

   g $(5x - 7)(6x - 1)$ **h** $2(4x - 5)(3x - 4)$ **i** $4(3x - 5)(7x - 4)$

MANIPULATING ALGEBRAIC EXPRESSIONS

EXERCISE 20 – FACTORISING QUADRATICS #7 ($a > 1$, $c < 0$)

Hint: some of the questions have a numerical factor as well as the algebraic ones. Take the number out first.

Factorise the following quadratics fully.

1. **a** $2a^2 - a - 1$ **b** $2g^2 + g - 1$ **c** $4r^2 + r - 3$

d $3v^2 - 4v - 4$ **e** $2x^2 - x - 6$ **f** $3h^2 + 5h - 12$

g $3k^2 - 5k - 12$ **h** $3b^2 + b - 10$ **i** $3x^2 - x - 10$

2. **a** $8b^2 - 6b - 5$ **b** $4s^2 + 4s - 15$ **c** $6m^2 + m - 12$

d $12w^2 + 10w - 8$ **e** $10n^2 - 9n - 9$ **f** $10c^2 + 9c - 9$

g $10d^2 + 19d - 15$ **h** $9p^2 - 4$ **i** $18t^2 - 21t - 4$

3. **a** $32z^2 - 50$ **b** $21e^2 + 23e - 20$ **c** $108y^2 - 147$

d $24q^2 + 18q - 15$ **e** $14u^2 + 31u - 10$ **f** $24f^2 + 14f - 24$

g $72x^2 - 98$ **h** $12n^2 + 8n - 15$ **i** $t^4 - 81$

EXERCISE 21* – FACTORISING QUADRATICS #8 (MIXED EXERCISE WHERE $a > 1$)

Hint: some of the questions have a numerical factor as well as the algebraic ones. Take out the number first.

Factorise the following quadratics fully.

1. **a** $6g^2 - 19g + 10$ **b** $4p^2 - 9$ **c** $12a^2 + 8a + 1$

d $21u^2 - 23u - 20$ **e** $18h^2 + 15h + 2$ **f** $36q^2 - 60q + 25$

g $9i^2 - 15i - 36$ **h** $12b^2 - 28b + 15$ **i** $10v^2 - 29v + 10$

2. **a** $12r^2 - 14r - 20$ **b** $5j^2 - 18j - 8$ **c** $20e^2 + 23e + 6$

d $14w^2 + 9w + 1$ **e** $3y^2 + 2y - 1$ **f** $12k^2 - k - 6$

g $4d^2 + 9d - 9$ **h** $45s^2 - 20$ **i** $4x^2 + 5x + 1$

3. **a** $4z^2 + 16z + 15$ **b** $8m^2 - 6m + 1$ **c** $36e^2 - 64$

d $16n^2 + 12n - 10$ **e** $9f^2 - 12f + 4$ **f** $20t^2 + 24t - 9$

g $12x^2 + 8x - 15$ **h** $a^2x^2 + 2abx + b^2$ **i** $a^2x^2 - b^2$

EXERCISE 20 – FACTORISING QUADRATICS #6 ($a > 1$, SIGNS DIFFERENT) – ANSWERS

1. **a** $(2a + 1)(a - 1)$ **b** $(g + 1)(2g - 1)$ **c** $(r + 1)(4r - 3)$

   d $(3v + 2)(v - 2)$ **e** $(2x + 3)(x - 2)$ **f** $(h + 3)(3h - 4)$

   g $(3k + 4)(k - 3)$ **h** $(b + 2)(3b - 5)$ **i** $(3x + 5)(x - 2)$

2. **a** $(2b + 1)(4b - 5)$ **b** $(2s + 5)(2s - 3)$ **c** $(2m + 3)(3m - 4)$

   d $2(3w + 4)(2w - 1)$ **e** $(5n + 3)(2n - 3)$ **f** $(2c + 3)(5c - 3)$

   g $(2d + 5)(5d - 3)$ **h** $(3p + 2)(3p - 2)$ **i** $(6t + 1)(3t - 4)$

3. **a** $2(4z + 5)(4z - 5)$ **b** $(3e + 5)(7e - 4)$ **c** $3(6y + 7)(6y + 7)$

   d $3(4q + 5)(2q - 1)$ **e** $(2u + 5)(7u - 2)$ **f** $2(3f + 4)(4f - 3)$

   g $2(6x - 7)(6x + 7)$ **h** $(6n - 5)(2n + 3)$ **i** $(t - 3)(t + 3)(t^2 + 9)$

EXERCISE 21* – FACTORISING QUADRATICS #7 (MIXED EXERCISE) – ANSWERS

1. **a** $(2g - 5)(3g - 2)$ **b** $(2p + 3)(2p - 3)$ **c** $(6a + 1)(2a + 1)$

   d $(3u - 5)(7u + 4)$ **e** $(6h + 1)(3h + 2)$ **f** $(6q - 5)^2$

   g $3(3i + 4)(i - 3)$ **h** $(6b - 5)(2b - 3)$ **i** $(2v - 5)(5v - 2)$

2. **a** $2(6r + 5)(r - 2)$ **b** $(5j + 2)(j - 4)$ **c** $(5e + 2)(4e + 3)$

   d $(7w + 1)(2w + 1)$ **e** $(y + 1)(3y - 1)$ **f** $(3k + 2)(4k - 3)$

   g $(d + 3)(4d - 3)$ **h** $5(3s + 2)(3s - 2)$ **i** $(4x + 1)(x + 1)$

3. **a** $(2z + 3)(2z + 5)$ **b** $(2m - 1)(4m - 1)$ **c** $4(3z + 4)(3z - 4)$

   d $2(4n + 5)(2n - 1)$ **e** $(3f - 2)^2$ **f** $(2t + 3)(10t - 3)$

   g $(6x - 5)(2x + 3)$ **h** $(ax + b)^2$ **i** $(ax - b)(ax + b)$

EXERCISE 22 – COMPLETING THE SQUARE ($a = 1$)

Q1 – 12 $a = 1$, b is even.

Q13 – 24 $a = 1$, b is either odd, a fraction or a variable.

Write the following quadratic expressions in completed square form.

1. a $k^2 + 2k$ b $a^2 - 2a$ c $r^2 + 4r$

    d $d^2 - 4d$ e $j^2 + 6j$ f $v^2 - 6v$

    g $i^2 + 2i + 1$ h $c^2 + 6c - 1$ i $b^2 - 4b + 3$

2. a $x^2 + 8x - 7$ b $x^2 - 10x + 5$ c $x^2 + 12x - 9$

    d $x^2 + 3x$ e $x^2 + x$ f $x^2 - 5x$

    g $x^2 - 7x$ h $x^2 - x + 3$ i $x^2 + 9x + 2$

3. a $x^2 - 11x - 7$ b $x^2 + bx$ c $x^2 + bx + c$

    d $x^2 + \dfrac{1}{2}x$ e $x^2 - \dfrac{3}{2}x$ f $x^2 - \dfrac{7}{2}x$

    g $x^2 + \dfrac{1}{2}x - \dfrac{1}{4}$ h $x^2 - \dfrac{3}{2}x + \dfrac{5}{4}$ i $x^2 - \dfrac{7}{2}x + \dfrac{49}{16}$

EXERCISE 23* – COMPLETING THE SQUARE #2 ($a \neq 1$)

Write the following expressions in completed square form as simply as possible.

1. a $2x^2 - 8x$ b $4x^2 - 20x$ c $5x^2 + 20x$

    d $3x^2 - 12x$ e $7x^2 + 21x$ f $-x^2 - 3x$

    g $-2x^2 - 8x$ h $-6x^2 - 12x$ i $x - 3x^2$

2. a $2x^2 - 4x - 8$ b $5x^2 + 15x - 5$ c $3x^2 + 9x - 1$

    d $2x^2 - 3x - 5$ e $3x^2 + 6x + 4$ f $5x^2 - 3x - 1$

    g $3x^2 - 9x - 2$ h $5x^2 + 12x - 3$ i $3x^2 + 7x + 2$

3. a $2 - 4x - x^2$ b $2 - 3x - 5x^2$ c $-2x^2 + 4x - 5$

    d $-6 - 3x^2 + 5x$ e $2 - 3x + 7x^2$ f $-x - 3x^2 - 2$

    e $7x - 6 - 2x^2$ f $-7 - 3x^2 - 8x$ g $-5x - 1 - 4x^2$

MANIPULATING ALGEBRAIC EXPRESSIONS

EXERCISE 22 – COMPLETING THE SQUARE ($a = 1$) – ANSWERS

1.
a $(k + 1)^2 - 1$
b $(a - 1)^2 - 1$
c $(r + 2)^2 - 4$

d $(d - 2)^2 - 4$
e $(j + 3)^2 - 9$
f $(v - 3)^2 - 9$

g $(i + 1)^2$
h $(c + 3)^2 - 10$
i $(b - 2)^2 - 1$

2.
a $(x + 4)^2 - 23$
b $(q - 5)^2 - 20$
c $(u + 6)^2 - 45$

d $(x + \frac{3}{2})^2 - \frac{9}{4}$
e $(x + \frac{1}{2})^2 - \frac{1}{4}$
f $(x + \frac{5}{2})^2 - \frac{25}{4}$

g $(x - \frac{7}{2})^2 - \frac{49}{4}$
h $(x - \frac{1}{2})^2 + \frac{11}{4}$
i $(x + \frac{9}{2})^2 - \frac{73}{4}$

3.
a $(x - \frac{11}{2})^2 - \frac{149}{4}$
b $(x + \frac{b}{2})^2 - \frac{b^2}{4}$
c $(x + \frac{b}{2})^2 - \frac{b^2 - 4c}{4}$

d $(x + \frac{1}{4})^2 - \frac{1}{16}$
e $(x - \frac{3}{4})^2 - \frac{9}{16}$
f $(x - \frac{7}{4})^2 - \frac{49}{16}$

g $(x + \frac{1}{4})^2 - \frac{5}{16}$
h $(x - \frac{3}{4})^2 + \frac{11}{16}$
i $(x - \frac{7}{4})^2$

EXERCISE 23* – COMPLETING THE SQUARE ($a > 1$) – ANSWERS

1.
a $2(x - 2)^2 - 8$
b $4(x - \frac{5}{2})^2 - 25$
c $5(x + 2)^2 - 20$

d $3(x - 2)^2 - 12$
e $7(x + \frac{3}{2})^2 - \frac{63}{4}$
f $\frac{9}{4} - (x + \frac{3}{2})^2$

g $8 - 2(x + 2)^2$
h $6 - 6(x + 1)^2$
i $\frac{1}{12} - 3(x - \frac{1}{6})^2$

2.
a $2(x - 1)^2 - 10$
b $5(x + \frac{3}{2})^2 - \frac{65}{4}$
c $3(x + \frac{3}{2})^2 - \frac{31}{4}$

d $2(x - \frac{3}{4})^2 - \frac{49}{8}$
e $3(x + 1)^2 + 1$
f $5(x - \frac{3}{10})^2 - \frac{29}{20}$

g $3(x - \frac{3}{2})^2 - \frac{35}{4}$
h $5(x + \frac{6}{5})^2 - \frac{51}{5}$
i $3(x + \frac{7}{6})^2 - \frac{25}{12}$

3.
a $6 - (x + 2)^2$
b $\frac{49}{20} - 5(x + \frac{3}{10})^2$
c $-2(x - 1)^2 - 3$

d $-3(x - \frac{5}{6})^2 - \frac{47}{12}$
e $7(x - \frac{3}{14})^2 + \frac{47}{28}$
f $-3(x + \frac{1}{6})^2 - \frac{23}{12}$

g $\frac{1}{8} - 2(x - \frac{7}{4})^2$
h $-3(x + \frac{4}{3})^2 - \frac{5}{3}$
i $\frac{9}{16} - 4(x + \frac{5}{8})^2$

EXERCISE 24 – SIMPLIFYING ALGEBRAIC FRACTIONS

Hint: As a general rule, you should always factorise expressions which can be factorised before doing anything else.

Simplify the following algebraic fractions.

1. $\dfrac{3x}{3}$

2. $\dfrac{4p}{2}$

3. $\dfrac{9q}{6}$

4. $\dfrac{4s}{16s}$

5. $\dfrac{x^2}{x}$

6. $\dfrac{3t^3}{6t}$

7. $\dfrac{y^2}{y^3}$

8. $\dfrac{5a^4}{10a^2}$

9. $\dfrac{6t^3}{8t^4}$

10. $\dfrac{s}{-s}$

11. $\dfrac{3p}{-2p}$

12. $\dfrac{-6n}{4n^3}$

13. $\dfrac{8p^3}{-2p^2}$

14. $\dfrac{3p+6}{3}$

15. $\dfrac{4y-6}{2}$

16. $\dfrac{x-3}{-1}$

17. $\dfrac{s-4}{4-s}$

18. $\dfrac{3}{6q+9}$

19. $\dfrac{(r-2)(r+2)}{r-2}$

20. $\dfrac{(a+3)(a+4)}{a+4}$

21. $\dfrac{3(i+2)}{i+2}$

22. $\dfrac{3(n-4)}{4(n-4)}$

23. $\dfrac{(b+5)(b-2)}{3b+15}$

24. $\dfrac{12(x-3)}{9(x-3)}$

25. $\dfrac{12c-8}{4}$

26. $\dfrac{2h-6}{3-h}$

27. $\dfrac{2g-4}{3g-6}$

28. $\dfrac{6p+3}{10p+5}$

29. $\dfrac{e^2-3e}{e}$

30. $\dfrac{2j^2+5j}{2j}$

31. $\dfrac{3d^2-7d}{4d}$

32. $\dfrac{p-1}{1-p}$

33. $\dfrac{f^2-1}{f-1}$

34. $\dfrac{6-5m}{5m-6}$

35. $\dfrac{12q-18r}{15r-10q}$

36. $\dfrac{k^2+2k+1}{(k+1)^2}$

37. $\dfrac{m^2+3m+2}{m^2-4}$

38. $\dfrac{(q+3)(q+4)}{3q+12}$

39. $\dfrac{4x(x-7)}{2x(x+3)}$

40. $\dfrac{r^2+r-6}{r^2-r-2}$

41. $\dfrac{t^2-3t-28}{t^2+7t+12}$

42. $\dfrac{27p^2-48}{9p-12}$

43. $\dfrac{2u^2+3u+1}{2u^2-u-1}$

44. $\dfrac{2n^2+5n-12}{2n^2-n-3}$

45. $\dfrac{12v^2+10v-12}{18-15v-18v^2}$

46. $\dfrac{16t^2-9}{9-16t^2}$

47. $\dfrac{25s^2-4}{15s^2+4s-4}$

48. $\dfrac{5z^3-15z+10}{21z-7z^3-14}$

EXERCISE 24 – SIMPLIFYING ALGEBRAIC FRACTIONS – ANSWERS

1. x

2. $2p$

3. $\frac{3q}{2}$

4. $\frac{1}{4}$

5. x

6. $\frac{t^2}{2}$

7. $\frac{1}{y}$

8. $\frac{a^2}{2}$

9. $\frac{3}{4t}$

10. -1

11. $-\frac{3}{2}$

12. $-\frac{3}{2n^2}$

13. $-4p$

14. $p+2$

15. $2y-3$

16. $3-x$

17. -1

18. $\frac{1}{2q+3}$

19. $r+2$

20. $a+3$

21. 3

22. $\frac{3}{4}$

23. $\frac{b-2}{3}$

24. $\frac{4}{3}$

25. $3c-2$

26. -2

27. $\frac{2}{3}$

28. $\frac{3}{5}$

29. $e-3$

30. $\frac{1}{2}(2j+5)$

31. $\frac{1}{4}(3d-7)$

32. -1

33. $f+1$

34. -1

35. $-\frac{6}{5}$

36. 1

37. $\frac{m+1}{m-2}$

38. $\frac{q+3}{3}$

39. $\frac{2(x-7)}{x+3}$

40. $\frac{r+3}{r+1}$

41. $\frac{t-7}{t+3}$

42. $3p+4$

43. $\frac{u+1}{u-1}$

44. $\frac{n+4}{n+1}$

45. $-\frac{2}{3}$

46. -1

47. $\frac{5s+2}{3s+2}$

48. $-\frac{5}{7}$

EXERCISE 25 – ADDING & SUBTRACTING ALGEBRAIC FRACTIONS

Questions 1 – 4 are a little warm up to make sure you remember how to add and subtract fractions.

Combine and simplify the following.

1. $\dfrac{1}{3}+\dfrac{1}{4}$

2. $\dfrac{2}{5}+\dfrac{3}{8}$

3. $\dfrac{5}{6}-\dfrac{4}{9}$

4. $\dfrac{6}{11}-\dfrac{2}{7}$

5. $\dfrac{1}{b}+\dfrac{2}{b}$

6. $\dfrac{4}{q}-\dfrac{1}{q}$

7. $\dfrac{1}{f^2}+\dfrac{5}{f^2}$

8. $\dfrac{7}{h+1}-\dfrac{3}{h+1}$

9. $\dfrac{4y}{c^2}-\dfrac{y}{c^2}$

10. $\dfrac{2}{3j}+\dfrac{3}{2j}$

11. $\dfrac{3}{5m}-\dfrac{1}{4m}$

12. $\dfrac{2x}{3y}-\dfrac{x}{4y}$

13. $\dfrac{2}{5e}+\dfrac{3}{10e}-\dfrac{4}{15e}$

14. $\dfrac{1}{a+1}+\dfrac{1}{a+2}$

15. $\dfrac{1}{p+1}-\dfrac{1}{p-1}$

16. $\dfrac{2}{s+2}+\dfrac{3}{s+1}$

17. $\dfrac{4}{g-3}-\dfrac{2}{g+1}$

18. $\dfrac{2}{d+1}-\dfrac{2}{d+3}$

19. $\dfrac{3i}{i-2}-\dfrac{i}{i+2}$

20. $\dfrac{3}{(k+4)^2}+\dfrac{4}{k+4}$

21. $\dfrac{4n}{(n-3)^2}+\dfrac{n}{n-3}$

22. $\dfrac{2}{q^2+3q+2}-\dfrac{2}{q+2}$

23. $\dfrac{7}{2-s}+\dfrac{3}{(2-s)(3+s)}$

24. $\dfrac{1}{u^2+u-12}+\dfrac{1}{u^2+2u-8}$

EXERCISE 25 – ADDING AND SUBTRACTING ALGEBRAIC FRACTIONS – ANSWERS

1. $\frac{7}{12}$

2. $\frac{31}{40}$

3. $\frac{7}{18}$

4. $\frac{20}{77}$

5. $\frac{3}{b}$

6. $\frac{3}{q}$

7. $\frac{6}{f^2}$

8. $\frac{4}{h+1}$

9. $\frac{3y}{c^2}$

10. $\frac{13}{6j}$

11. $\frac{7}{20m}$

12. $\frac{5x}{12y}$

13. $\frac{13}{30e}$

14. $\frac{2a+3}{(a+1)(a+2)}$

15. $\frac{2}{1-p^2}$

16. $\frac{5s+8}{(s+2)(s+1)}$

17. $\frac{2(g+5)}{(g-3)(g+1)}$

18. $\frac{4}{(d+1)(d+3)}$

19. $\frac{2i(i+4)}{i^2-4}$

20. $\frac{4k+19}{(k+4)^2}$

21. $\frac{n(n+1)}{(n-3)^2}$

22. $\frac{-2q}{(q+2)(q+1)}$

23. $\frac{24+7s}{(2-s)(3+s)}$

24. $\frac{2u-5}{(u+4)(u-3)(u-2)}$

EXERCISE 26 – MULTIPLYING & DIVIDING ALGEBRAIC FRACTIONS

Hint: Remember that $x + 2$ is NOT divisible by either 2 or x.

Combine and simplify the following.

1. $\dfrac{x}{2} \times \dfrac{x}{3}$

2. $\dfrac{2b}{3} \times \dfrac{b}{2}$

3. $\dfrac{3p}{2} \times \dfrac{4q}{9}$

4. $\dfrac{2}{f} \times \dfrac{f}{4}$

5. $\dfrac{n}{5} \times \dfrac{15}{2n}$

6. $\dfrac{3c}{10} \times \dfrac{5}{2c}$

7. $\dfrac{q^2}{3} \times \dfrac{6}{q}$

8. $\dfrac{4a}{j} \times \dfrac{6j}{3a}$

9. $\dfrac{r+1}{3} \times \dfrac{6}{r+1}$

10. $\dfrac{2e-3}{e} \times \dfrac{e}{e-3}$

11. $\dfrac{y^2+3y+2}{y} \times \dfrac{2y}{(y+1)^2}$

12. $\dfrac{d-3}{3} \times \dfrac{6}{2d-6}$

13. $\dfrac{u}{2} \div \dfrac{u}{4}$

14. $\dfrac{1}{s} \div \dfrac{3}{s}$

15. $\dfrac{2}{3g} \div \dfrac{6}{g}$

16. $\dfrac{3xy}{x} \div \dfrac{3x}{y}$

17. $\dfrac{3}{h-2} \div \dfrac{2h}{3h-6}$

18. $\dfrac{3m^3}{(m+5)^2} \div \dfrac{2m^2}{m^2+7m+10}$

19. $\dfrac{2k+8}{3k} \div \dfrac{k+4}{3k}$

20. $\dfrac{x^2-4}{x+2} \div \dfrac{x-2}{3}$

21. $\dfrac{(t-4)^2}{2t} \div \dfrac{t-4}{6t}$

22. $\dfrac{2y+3}{y-3} \div \dfrac{2y^2+y-3}{3y^2-7y-6}$

23. $\dfrac{3i+4}{i+1} \div \dfrac{9i+12}{(i+1)^2}$

24. $\dfrac{a^2+4a+3}{a^2-3a-4} \div \dfrac{a^2+a-6}{a^2-6a+8}$

EXERCISE 26 – MULTIPLYING AND DIVIDING ALGEBRAIC FRACTIONS

1. $\dfrac{x^2}{6}$

2. $\dfrac{b^2}{3}$

3. $\dfrac{2pq}{3}$

4. $\dfrac{1}{2}$

5. $\dfrac{3}{2}$

6. $\dfrac{3}{4}$

7. $2q$

8. 8

9. 2

10. $\dfrac{2e-3}{e-3}$

11. $\dfrac{2(y+2)}{y+1}$

12. 1

13. 2

14. $\dfrac{1}{3}$

15. $\dfrac{1}{9}$

16. $\dfrac{y^2}{x}$

17. $\dfrac{9}{2h}$

18. $\dfrac{3m(m+2)}{2(m+5)}$

19. 2

20. 3

21. $3(t-4)$

22. $\dfrac{3y+2}{y-1}$

23. $\dfrac{i+1}{3}$

24. 1

EXERCISE 27 – MIXED ALGEBRAIC FRACTIONS PROBLEMS

Simplify the following expressions.

1. $\dfrac{3}{i^2+2i}-\dfrac{1}{i}$

2. $\dfrac{2d^2}{6d}$

3. $\dfrac{3}{r+2}+\dfrac{4}{(r+2)^2}$

4. $\dfrac{4}{h^2-4}+\dfrac{2}{h-2}$

5. $\dfrac{s^2+s-6}{2s^2-s-6}$

6. $\dfrac{f+2}{3}\times\dfrac{12}{2f+4}$

7. $\dfrac{3e^2}{4y}\div\dfrac{9e}{2y^2}$

8. $\dfrac{c+3}{c+4}+\dfrac{c+3}{c^2+7c+12}$

9. $\dfrac{q}{q^2+3q+2}-\dfrac{2}{q+2}$

10. $\dfrac{2}{t+1}+\dfrac{1}{t+2}$

11. $\dfrac{6y^2-y-12}{10y^2-11y-6}$

12. $\dfrac{2b}{(b-2)(b+2)}-\dfrac{2}{b+2}$

13. $\dfrac{3}{(s+3)(s+2)}+\dfrac{2s}{s+3}$

14. $\dfrac{1}{(a+3)^2}-\dfrac{4}{a+3}$

15. $\dfrac{6g}{2g-5}\times\dfrac{6g-15}{3g}$

16. $\dfrac{5}{(k-1)(k+1)}+\dfrac{2}{k+1}$

17. $\dfrac{2}{(j+4)(j-3)}+\dfrac{3}{j-3}$

18. $\dfrac{n^3}{3n-4}\div\dfrac{4n^2}{9n-12}$

19. $\dfrac{r^2-4}{r+3}\times\dfrac{r^2-9}{r-2}$

20. $\dfrac{5}{(m+2)(m+3)}-\dfrac{2}{m+3}$

21. $\dfrac{7p+2}{p^2+p-6}-\dfrac{2}{p-2}$

22. $\dfrac{t+2}{t+3}+\dfrac{t+2}{t^2+5t+6}$

23. $\dfrac{2}{2n^2-n-6}+\dfrac{1}{2n^2+n-3}$

24. $\dfrac{x-2}{x^2+5x+6}+\dfrac{x}{2x+6}$

EXERCISE 27 – MIXED EXERCISE ON ALGEBRAIC FRACTIONS – ANSWERS

1. $\dfrac{1-i}{i(i+2)}$

2. $\dfrac{d}{3}$

3. $\dfrac{3r+10}{(r+2)^2}$

4. $\dfrac{2}{h-2}$

5. $\dfrac{s+3}{2s+3}$

6. 2

7. $\dfrac{ey}{6}$

8. 1

9. $\dfrac{-1}{q+1}$

10. $\dfrac{3t+5}{(t+1)(t+2)}$

11. $\dfrac{3y+4}{5y+2}$

12. $\dfrac{4}{b^2-4}$

13. $\dfrac{2s^2+4s+3}{(s+3)(s+2)}$

14. $-\dfrac{4a+11}{(a+3)^2}$

15. 6

16. $\dfrac{2k+3}{k^2-1}$

17. $\dfrac{3j+14}{(j+4)(j-3)}$

18. $\dfrac{3n}{4}$

19. $(r+2)(r-3)$

20. $\dfrac{1-2m}{(m+2)(m+3)}$

21. $\dfrac{5p-4}{(p-2)(p+3)}$

22. 1

23. $\dfrac{3n-4}{(2n+3)(n-2)(n-1)}$

24. $\dfrac{x^2+4x-4}{2(x+2)(x+3)}$